

# MODEL PAPER-2

B.A., B.Sc. Mathematics, Semester - III

## Laplace Transforms

Time : 3 Hours ]

[ Max : 75 Marks

**Section A** ( $5 \times 5 = 25$  Marks)

Answer any **Five** of the following questions.

Each question carries **Five** marks.

1. Define Integral Transform.
2. Show that  $t^2$  is f exponential order 3.
3. Show that :

$$L[1 + te^{-t}]^3 = \frac{1}{s} + \frac{3}{(s+1)^2} + \frac{6}{(s+2)^3} + \frac{6}{(s+3)^4}.$$

4. Find the Laplace Transform of  $f(t)$  defined as

$$f(t) = \begin{cases} \frac{t}{\tau}, & \text{when } 0 < t < \tau. \\ 1, & \text{when } t > \tau. \end{cases}$$

5. Prove that  $L\{j_1(t)\} = 1 - \frac{p}{\sqrt{p^2 + 1}}$ .
6. State and prove First Shifting theorem on Inverse Laplace Transform.
7. Define Heavi-side's expansion formula and using Heavi-side's expansion formula find

$$(i) \quad L^{-1} \left[ \frac{2s^2 + 5s - 4}{s^3 + s^2 - 2s} \right]$$

$$(ii) \quad L^{-1} \left[ \frac{s^2 - 6}{s^3 + 4s^2 + 3s} \right]$$



$$(iii) L^{-1} \left[ \frac{3s + 1}{(s - 1)(s^2 + 1)} \right].$$

8. Find the Inverse Laplace Transform of

$$(i) \frac{s + 2}{(s^2 + 4s + 5)^2} \quad (ii) \frac{s + 3}{(s^2 + 6s + 13)^2}.$$

**Section B** ( $5 \times 10 = 50$  Marks)

Answer any **Five** of the following questions.

Each question carries **Ten** marks.

9. (a) Find the Laplace Transform of the function

$$f(t) = \begin{cases} 2t, & 0 \leq t \leq 5 \\ 1, & t > 5 \end{cases}$$

(Or) (b) Find Laplace transform of (a)  $\sin^2(at)$ , (b)  $(t^2 + 1)^2$ .

10. (a) Find the Laplace Transform of

$$(i) t \cos at \quad (ii) t^2 \sin at \\ (iii) t^3 e^{-3t} \quad (iv) t e^{-t} \sin 3t.$$

(Or) (b) Find  $L \left( \sqrt{t} - \frac{1}{\sqrt{t}} \right)^3$ .

11. (a) Prove that  $L \left[ \frac{\sin t}{t} \right] = \tan^{-1} \frac{1}{s}$  and hence find  $L \left[ \frac{\sin at}{t} \right]$ .

Does the Laplace Transform of  $\frac{\cos at}{t}$  exist? Explain?

(Or) (b) Find  $L \{ \operatorname{erf} \sqrt{t} \}$  and hence prove that

$$L \{ t \operatorname{erf} (2\sqrt{t}) \} = \frac{3p + 8}{p^2 (p + 4)^{3/2}}.$$

12. (a) Find the Inverse Laplace Transform of

$$(i) \log \left( \frac{s + 1}{s - 1} \right) \quad (ii) \log \frac{s^2 + 1}{s(s + 1)}.$$

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(Or) (b) Evaluate

$$(i) \mathcal{L}^{-1} \left\{ \frac{3p-2}{p^2-4p+20} \right\} \quad (ii) \mathcal{L}^{-1} \left\{ \frac{3p+7}{p^2-2p-3} \right\}$$

13. (a) Find the Inverse Laplace Transform of  $\left[ \frac{1}{s(s+a)^3} \right]$

(Or) (b) Use convolution theorem to find

$$(i) \mathcal{L}^{-1} \left[ \frac{1}{s(s^2+4)^2} \right] \quad (ii) \mathcal{L}^{-1} \left[ \frac{s}{(s^2+4)^3} \right]$$