MODEL PAPER - III

W.E.F from 2023-2024

Time : 3 Hours] [Max. : 75 Marks

SECTION – A (5 × 5 = 25 Marks)

Answer any FIVE of the following questions.

1. Compute: i) $\Gamma(-1/2)$ ii) $\Gamma(-3/2)$ iii) $\Gamma(-5/2)$.
[Q.No. 18, Pg No. 9]

2. Evaluate $\int_0^2 x\,(8-x^3)^{1/3}\,dx$. [Q.No. 21, Pg No. 10]

3. Find the radius of convergence of the series $\sum \frac{2^n x^n}{n!}$.
[Q.No. 3, Pg No. 28]

4. Show that $x = 0$ is an ordinary point of $y'' - xy' + 2y = 0$.
[Q.No. 9, Pg No. 31]

5. State and Prove Generating function for $H_n(x)$.
[Q.No. 1, Pg No. 45]

6. Prove that $2xH_n(x) = 2n\,H_{n-1}(x) + H_{n+1}(x)$. [Q.No. 5, Pg No. 48]

7. Prove that $nP_n(x) = xP_n'(x) - P_{n-1}'(x)$. [Q.No. 2, Pg No. 63]

8. Prove that $\frac{d}{dx}[x^{-n}J_n(x)] = -x^{-n}J_{n+1}(x)$. [Q.No. 5, Pg No. 81]

SECTION – B (5 × 10 = 50 Marks)

Answer any FIVE of the following questions.

9. a) Prove that: i) $\beta\left(m,\frac{1}{2}\right) = 2^{2m-1}\beta(m,m)$

ii) $\Gamma(m)\Gamma\left(m+\frac{1}{2}\right) = \frac{\sqrt{\pi}}{2^{2m-1}}\Gamma(2m)$

[Q.No. 32, Pg No. 18]

(OR)

b) Prove that $\int_0^{(\pi/2)} \frac{d\theta}{\sqrt{\sin\theta}} \times \int_0^{\pi/2} \sqrt{\sin\theta}\;d\theta = \pi$.

[Q.No. 38, Pg No. 23]

10. a) Find the power series solution of the equation
$(x^2+1)y'' + xy' - xy = 0$ in powers of x(i.e. about $x = 0$).
[Q.No. 12, Pg No. 32]

(OR)

b) Show that $x = 0$ is a regular point of $(2x + x^3)y'' - y' - 6xy = 0$ and find its solution about $x = 0$. [Q.No. 19, Pg No. 40]

11. a) Define Hermite differential equation and find its solution. [Q.No. 13, Pg No. 52]

(OR)

b) Evaluate $\int_{-\infty}^{\infty} x e^{-x^2} H_n(x) H_m(x) dx$. [Q.No. 17, Pg No. 59]

12. a) Show that $\int_{-1}^{1} x P_n(x) P_{n-1}(x) dx = \frac{2n}{4n^2-1}$ [Q.No. 15, Pg No. 73]

(OR)

b) State and Prove Rodrigue's Formula for $P_n(x)$. [Q.No. 11, Pg No. 68]

13. a) Prove that $\int_0^x t\,\{J_n(t)\}^2 dt = \frac{x^2}{2}\{J_n^2(x) - J_{n-1}(x)J_{n+1}(x)\}$. [Q.No. 19, Pg No. 93]

(OR)

b) Prove that: i. $J_0' = -J_1$; ii. $J_2 = J_0'' - x^{-1}J_0'$ and iii. $J_2 - J_0 = 2J_0''$. [Q.No. 14, Pg No. 89]