MODEL PAPERS

MODEL PAPER - I

W.E.F from 2023-2024

Time : 3 Hours] [Max. : 75 Marks

SECTION – A (5 × 5 = 25 Marks)

Answer any FIVE of the following questions.

1. Prove that: $\beta(m,n) = \int_0^{\pi/2} (\sin^2\theta)^{m-1}(\cos^2\theta)^{n-1} 2\sin\theta\cos\theta\, d\theta$.
   [Q.No. 1, Pg No. 2]

2. Theorem: $(1-x^2)T_n'(x) = -nxT_n(x) + nT_{n-1}(x)$.
   [Q.No. 6, Pg No. 5]

3. Find the radius of convergence of the $\frac{x}{2} + \frac{1.3}{2.5}x^2 + \frac{1.3.5}{2.5.8}x^3 + \cdots$
   [Q.No. 4, Pg No. 28]

4. Determine whether x=0 is an ordinary point or regular singular point of differential equation $2x^2\frac{d^2y}{dx^2} + 7x(x+1)\frac{dy}{dx} - 3y = 0$.
   [Q.No. 7, Pg No. 30]

5. Prove that $H_n'(x) = 2nH_{n-1}(x), n \geq 1$. [Q.No. 4, Pg No. 48]

6. Prove that $H_n(x) = 2^n\left\{e^{\left(-\frac{1}{4}\frac{d^2}{dx^2}\right)}x^n\right\}$. [Q.No. 2, Pg No. 46]

7. Express $f(x) = x^4 + 3x^3 - x^2 + 5x - 2$ in terms of Legendre polynomials. [Q.No. 6, Pg No. 65]

8. Prove that $x.J_n'(x) = n.J_n(x) - x.J_{n+1}(x)$. [Q.No. 1, Pg No. 78]

SECTION – B (5 × 10 = 50 Marks)

Answer any FIVE of the following questions.

9. a) Derive Relation between beta and gamma functions.
   [Q.No. 28, Pg No. 13]

(OR)

b) Prove that $\int_0^1 \frac{x^2 dx}{\sqrt{(1-x^4)}} \times \int_0^1 \frac{dx}{\sqrt{(1+x^4)}} = \frac{\pi}{4\sqrt{2}}$. [Q.No. 31, Pg No. 16]

10. a) Find the series solution of $(1-x^2)y'' + 2y = 0, y(0) = 4,$ $y'(0) = 5$. [Q.No. 14, Pg No. 35]

(OR)

b) Apply power method to solve $y'' - y = x$. [Q.No. 18, Pg No. 39]

11. a) State and Prove Rodricues Formula for the Hermite Polynomials. [Q.No. 14, Pg No. 57]

(OR)

b) Prove that: i) $H_{2n}(0) = (-1)^n \frac{(2n)!}{n!}$ and ii) $H_{2n+1}(0) = 0$. [Q.No. 16, Pg No. 59]

12. a) Show that $\int_{-1}^{1} x^2 P_{n-1} P_{n+1} dx = \frac{2n(n+1)}{(2n-1)(2n+1)(2n+3)}$. [Q.No. 17, Pg No. 74]

(OR)

b) State and Prove Generating Function for $P_n(x)$. [Q.No. 12, Pg No. 69]

13. a) Show that [Q.No. 15, Pg No. 90]

i. $J_{-\frac{1}{2}}(x) = \sqrt{\left(\frac{2}{\pi x}\right)} . \cos x$, ii. $J_{\frac{1}{2}}(x) = \sqrt{\left(\frac{2}{\pi x}\right)} . \sin x$ and

iii. $\left[J_{\frac{1}{2}}(x)\right]^2 + \left[J_{-\frac{1}{2}}(x)\right]^2 = \frac{2}{\pi x}$

(OR)

b) Prove that $\sqrt{\left(\frac{\pi x}{2}\right)}\, J_{3/2}(x) = \frac{1}{x} \sin x - \cos x$. [Q.No. 17, Pg No. 92]